

MODELLING AIRBORNE FUNGAL SPORE
DATA WITH THE GAMMA DISTRIBUTION

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ABSTRACT

We present here a discussion on the performance of the gamma distribution as a model for airborne fungal spore daily concentrations, using data extracted from the Catalanian Aerobiological Network (Xarxa Aerobiològica de Catalunya, XAC). This is the sequel of a previous study, dealing with similar models for pollen concentrations (Worcester conference).

The gamma distribution has two parameters, a shape parameter and a scale parameter. The interpretation of these parameters and the relationship between them and the usual statistics are also discussed.

The study includes the eight spore types that are more abundant in the area covered by the network (*Alternaria*, *Arthrimum*, *Aspergillus/Penicillium*, *Cladosporium*, Coprinaceae, *Leptosphaeria*, *Ustilago* and Total spores).

For four of these spore types, the statistics are enough regular to allow a systematic statistical analysis. We examine here the distributions resulting from this analysis. The fit is assessed by means of a chi square statistic.

Figure 1. Mean daily concentration of Cladosporium (BCN/1999)

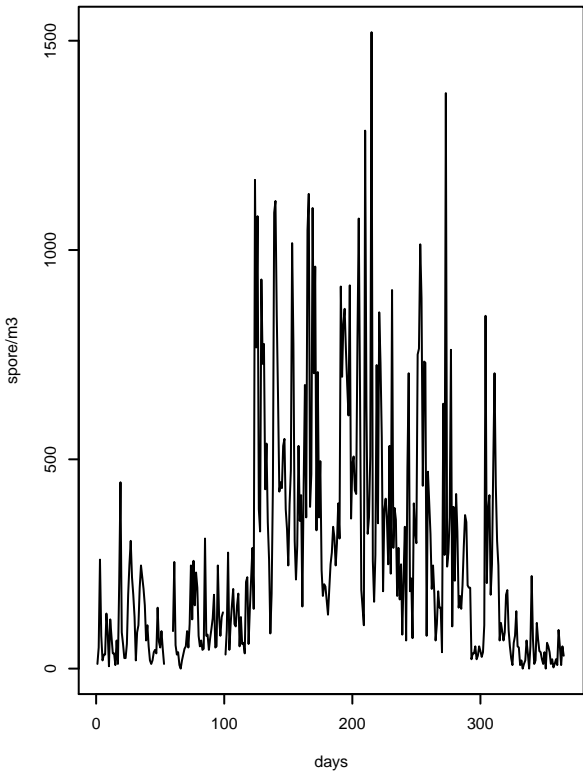
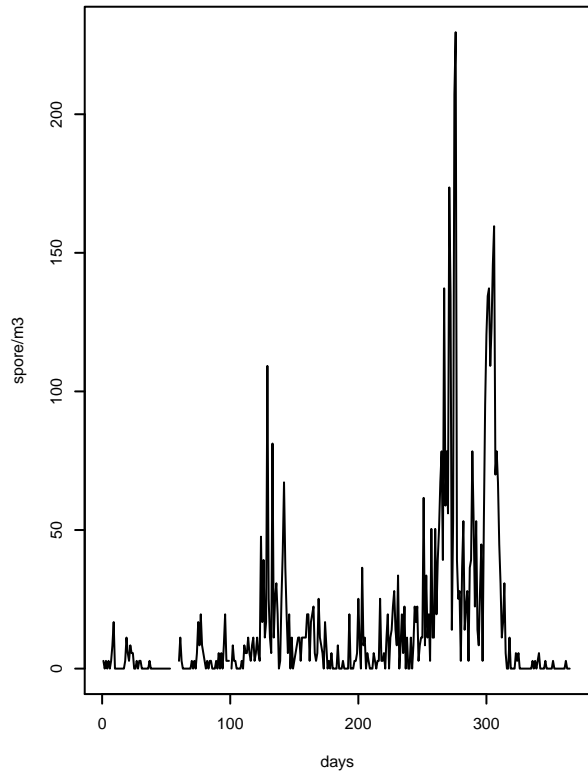


Figure 2. Mean daily concentration of Coprinaceae (BCN/1999)



GAMMA DISTRIBUTION

- Probability density function:

$$f(x) = C x^{\alpha-1} e^{-x/\beta}, \quad x > 0,$$

where C is a constant, which depends on α and β .

- Parameters:
 - * α is the shape parameter, no units, scale-free.
 - * β is the scale parameter, same units as the concentration.
- Relationship with the mean and variance:

$$\bar{x} = \alpha\beta,$$

$$s^2 = \alpha\beta^2,$$

$$CV = \frac{s}{\bar{x}} = \alpha^{-1/2}.$$

Figure 3. Gamma distributions with equal mean but different shape

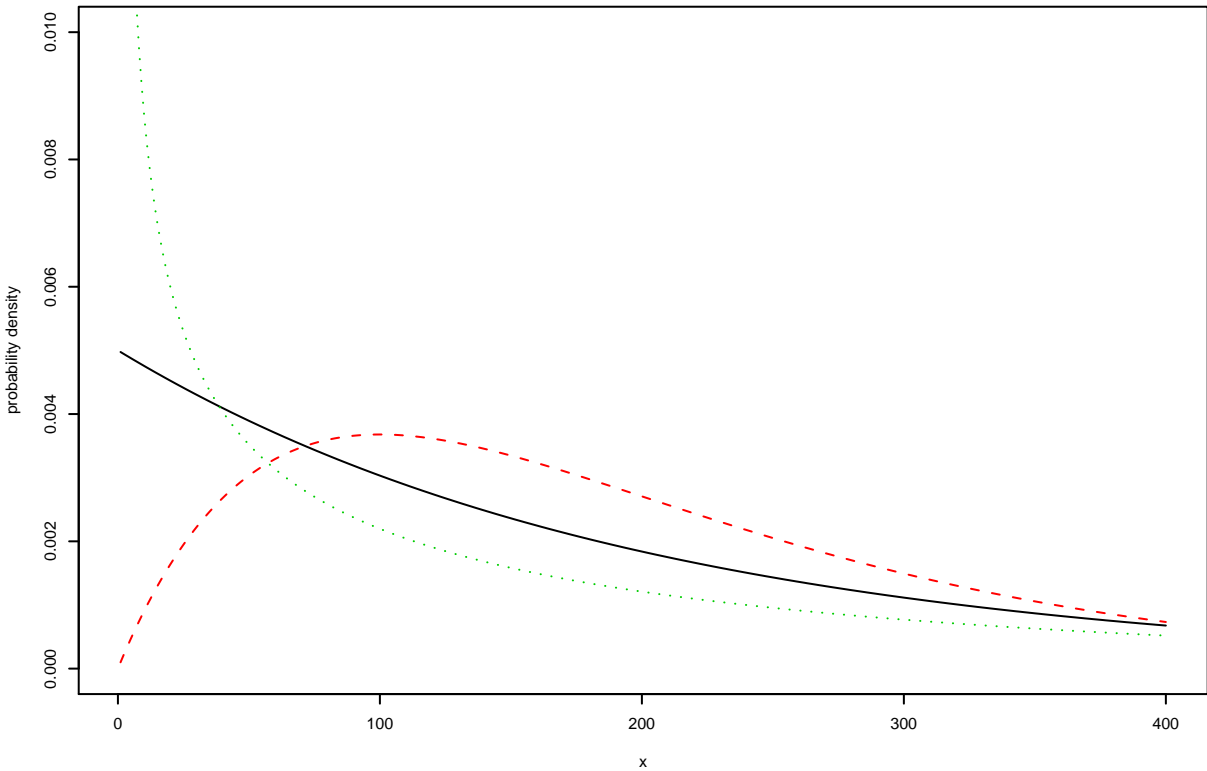


Figure 4. Mean daily concentration of Cladosporium (BCN/1999)

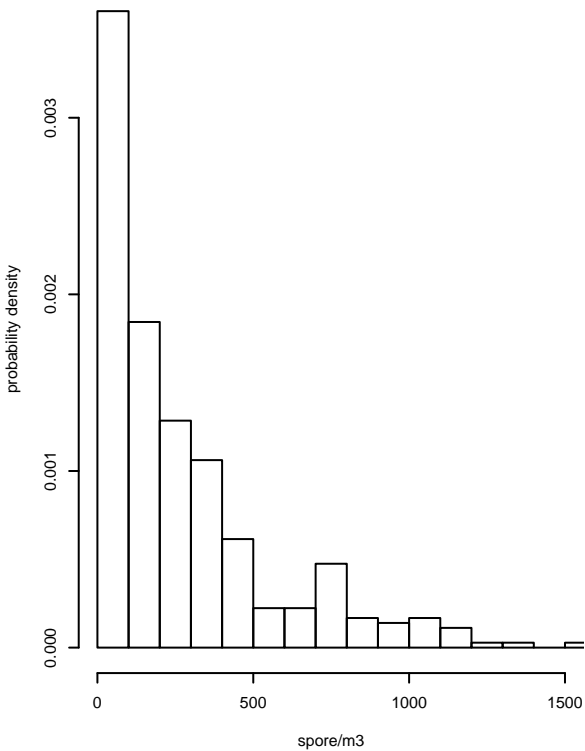


Figure 5. Gamma distribution fitted to Cladosporium data

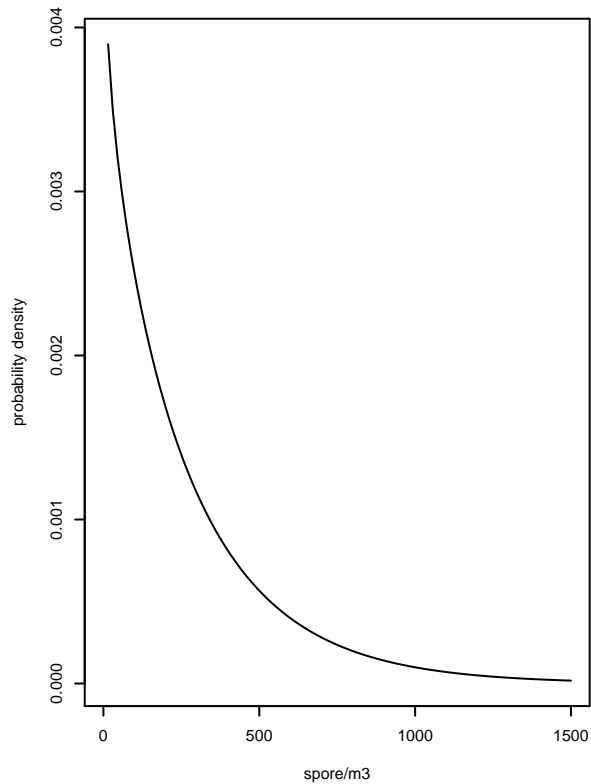


Figure 6. Mean daily concentration of Total spores (BCN/1999)

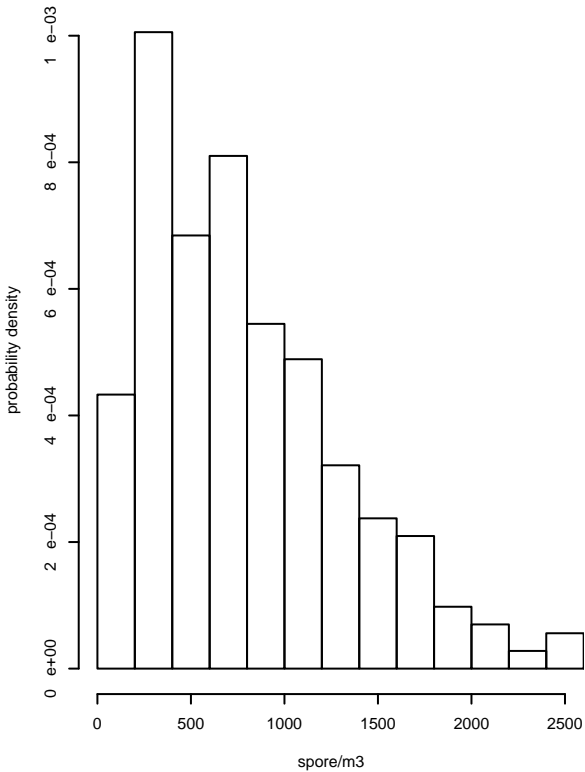
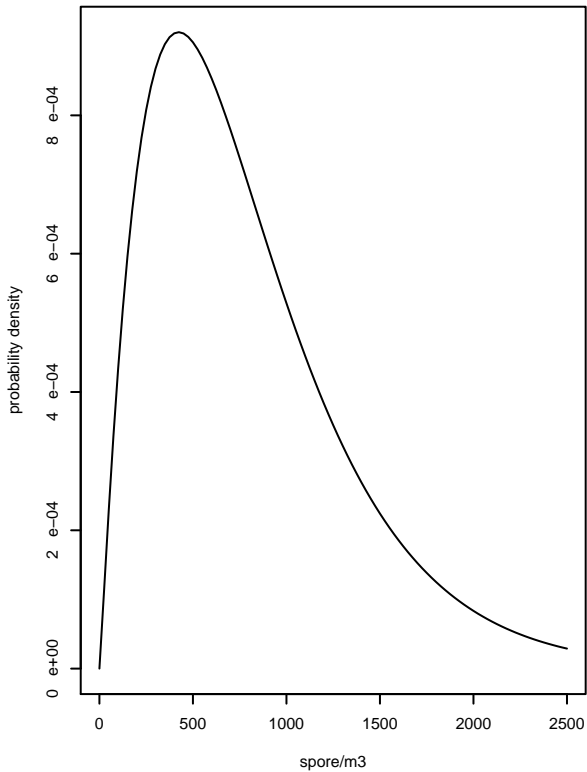


Figure 7. Gamma distribution fitted to Total spores data



EXAMPLE

For *Cladosporium* mean daily spore concentrations measured in the Barcelona station in 1999, the mean and the standard deviation are

$$\bar{x} = 271.69 \text{ spore/m}^3, \quad s = 283.85 \text{ spore/m}^3,$$

and the coefficient of variation is $CV = 1.045$.

Then the shape and scale parameters are

$$\alpha = \frac{(\bar{x})^2}{s^2} = \frac{1}{CV^2} = 0.9162,$$

$$\beta = \frac{s^2}{\bar{x}} = 296.6 \text{ spore/m}^3.$$

Note that α is dimensionless, while β , being a scale parameter, has the same units as the pollen concentrations.

ASSESSING THE FIT

- QQ plot:

Two-dimensional graphical display of the data set against the quantiles of a theoretical probability distribution.

The perfect fit corresponds to a plot where all the points belong to the line $y = x$.

- Chi square statistic:

Measure of the agreement between the probabilities given by a theoretical model, called the *expected proportions* and the proportions actually found in the data, called the *observed proportions*.

Based on a partition of the range of the variable into a set of intervals.

It can be used in a significance test, since it follows (approximately) a chi square distribution when the model fitted to the data is valid.

With 5 intervals, the critical value are $\chi_{0.05} = 7.81$ and $\chi_{0.01} = 11.34$.

Figure 8. Gamma QQ plot for Cladosporium

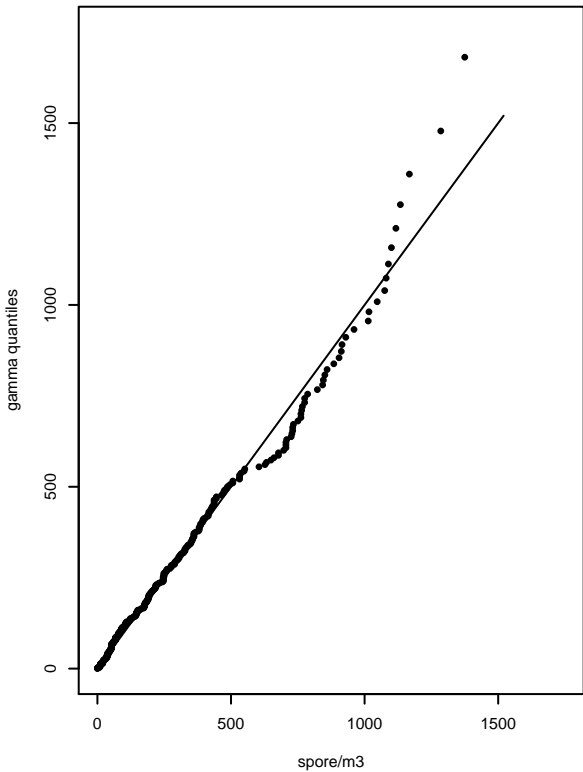


Figure 9. Gamma QQ plot for Cladosporium (sqrt scale)

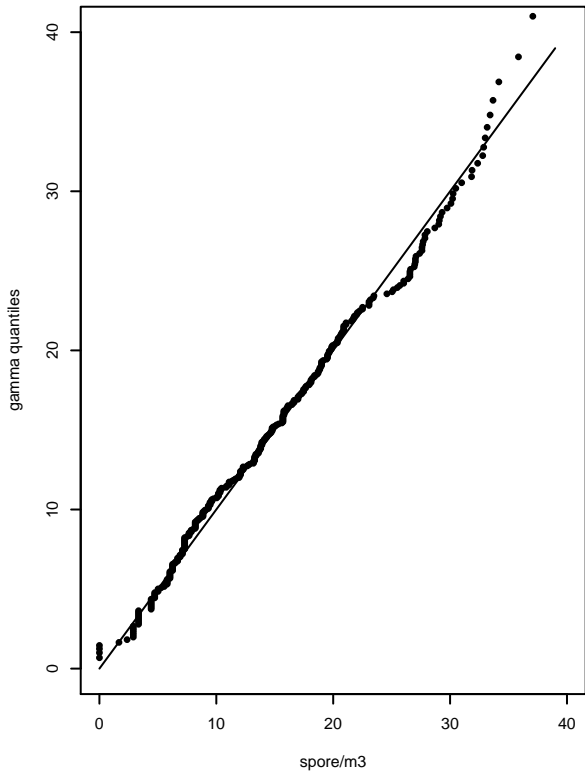
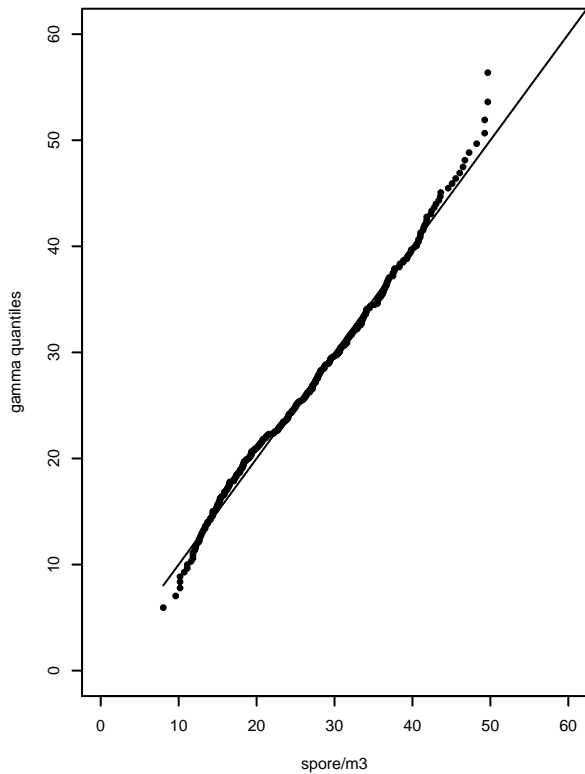


Figure 10. Gamma QQ plot for Total spores (sqrt scale)



CHI SQUARE STATISTICS (1)

TABLE 1. Chi square testing for *Cladosporium* (Bellaterra)

| Statistics | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
|---------------------------|--------|------|--------|--------|------|------|--------|--------|
| <u>Observed (%)</u> | | | | | | | | |
| sp/m ³ <50 | 2.6 | 10.1 | 4.9 | 10.7 | 12.8 | 11.6 | 6.8 | 1.6 |
| 50<s/m ³ <100 | 4.1 | 11.3 | 7.4 | 9.6 | 13.1 | 9.4 | 16.7 | 6.9 |
| 100<s/m ³ <250 | 26.5 | 28.7 | 20.6 | 24.8 | 21.5 | 20.5 | 22.7 | 24.2 |
| 250<s/m ³ <500 | 23.6 | 23.5 | 16.4 | 19.6 | 22.6 | 18.3 | 17.5 | 29.5 |
| s/m ³ >500 | 43.2 | 26.4 | 50.7 | 35.3 | 29.9 | 40.2 | 36.2 | 37.7 |
| <u>Expected (%)</u> | | | | | | | | |
| s/m ³ <50 | 10.1 | 13.3 | 9.5 | 19.9 | 14.3 | 9.9 | 17.3 | 8.7 |
| 50<s/m ³ <100 | 7.3 | 11.0 | 7.2 | 9.6 | 10.3 | 8.3 | 9.3 | 8.4 |
| 100<s/m ³ <250 | 16.7 | 24.9 | 16.9 | 18.4 | 22.5 | 19.8 | 18.6 | 21.5 |
| 250<s/m ³ <500 | 19.2 | 24.4 | 19.7 | 17.9 | 22.6 | 22.4 | 18.7 | 24.5 |
| s/m ³ >500 | 46.7 | 26.3 | 46.6 | 34.2 | 30.3 | 39.5 | 36.2 | 36.8 |
| Chi square | 48.8** | 4.66 | 14.3** | 24.0** | 3.41 | 4.49 | 48.4** | 26.9** |

CHI SQUARE STATISTICS (2)

TABLE 2. Chi square testing for Total spores (Bellaterra)

| Statistics | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
|-----------------------|------|------|--------|--------|------|------|------|------|
| <u>Observed (%)</u> | | | | | | | | |
| $s/m^3 < 500$ | 20.7 | 25.8 | 10.7 | 12.4 | 25.7 | 36.0 | 52.9 | 30.9 |
| $500 < s/m^3 < 1000$ | 28.5 | 19.1 | 17.8 | 30.3 | 28.2 | 26.3 | 20.3 | 24.8 |
| $1000 < s/m^3 < 1500$ | 15.0 | 21.4 | 11.5 | 20.9 | 19.0 | 15.5 | 12.6 | 19.0 |
| $1500 < s/m^3 < 2000$ | 13.3 | 13.0 | 17.5 | 13.2 | 9.8 | 7.8 | 4.1 | 10.5 |
| $s/m^3 > 2000$ | 22.5 | 20.6 | 42.5 | 23.1 | 17.3 | 14.4 | 10.1 | 14.9 |
| <u>Expected (%)</u> | | | | | | | | |
| $s/m^3 < 500$ | 22.5 | 24.3 | 13.3 | 24.9 | 27.3 | 37.8 | 50.2 | 30.4 |
| $500 < s/m^3 < 1000$ | 24.0 | 24.0 | 17.1 | 20.2 | 25.5 | 22.7 | 21.0 | 26.4 |
| $1000 < s/m^3 < 1500$ | 18.1 | 17.6 | 15.4 | 15.1 | 17.7 | 14.3 | 11.6 | 17.3 |
| $1500 < s/m^3 < 2000$ | 12.5 | 12.0 | 12.8 | 11.1 | 11.4 | 9.1 | 6.8 | 10.7 |
| $s/m^3 > 2000$ | 22.9 | 22.1 | 41.4 | 28.7 | 18.1 | 16.1 | 10.4 | 15.3 |
| Chi square | 5.46 | 7.28 | 12.3** | 54.3** | 2.69 | 4.09 | 4.78 | 1.01 |

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